Privacy-Preserving Cloud Database Querying

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Abstract

Due to its low cost, robustness, flexibility and ubiquitous nature, cloud computing is changing the way entities manage their data. However, various privacy concerns arise whenever potentially sensitive data is outsourced to the cloud.

This paper presents a novel approach for coping with such privacy concerns. The proposed scheme prevents the cloud server from learning any possibly sensitive plaintext in the outsourced databases. It also allows the database owner to delegate users to conducting content-level fine-grained private search and decryption. Moreover, our scheme supports private querying whereby neither the database owner nor the cloud server learns query details. Additional requirement that user's input be authorized by CA can also be supported.

Keywords: Privacy, Database, Cloud Computing, Attribute-based Encryption

1 Introduction

Cloud computing involves highly available massive compute and storage platforms offering a wide range of services. It provides a way to deliver application-as-a-service over the Internet. One of the most popular and basic cloud computing services is storage-as-a-service (SAAS). Initially deployed as a means of helping Internet giants cope with explosive user demand, SAAS, in place of traditional storage infrastructure, is shaping the way organizations manage their data. SAAS's low cost and performance fuels the success of cloud computing pioneers, such as Amazon and Google. For example, Amazon has been offering its "S3 storage service" for several years, while Google began offering "Google Storage for Developers" in Spring 2010. As a value-add layer over the simple storage service, some cloud-computing companies provide cloud database management systems to facilitate data maintenance, e.g. Amazon SimpleDB.

From the customer's point of view, there are several benefits of SAAS. First, the cost is very low. Instead of the outmoded storage-area network (SAN) concept, SAAS can be provided by a mesh of interconnected Linux servers with cheap disks managed by a distributed filesystem. Recent studies show that the cost per GB of enterprise SAN storage is around \$20/GB, whereas, cloud computing brings the cost down to as low as \$1/GB [25]. Second, storage maintenance is significantly simpler. Professional SAAS providers offer customer service (via phone and Internet) on a 24/7/365 basis. Third, storage access is robust and easy: SAAS is based on data centers around the globe and data can be accessed anywhere/anytime over the Internet. Data is geographically replicated and customers do not need to worry about data loss due to catastrophes. Finally, storage is available on demand. Storage space can be dynamically allocated thus eliminating the need for customers to plan ahead.

We consider two examples to demonstrate SAAS application scenarios. The first is health care providers (e.g. hospitals) that maintain databases of patient medical records. Outsourcing these databases to a cloud server can be a major step towards cutting health care cost as part of the Health Care Reform

Act [4]. The second example is educational institutions (e.g., universities) that maintain records of all current and past students. Universities confronted with financial pressures have the incentive to cut costs by outsourcing student databases to cloud servers. In these two examples, health care providers and educational institutions are *database owners*. Their constituent entities (e.g., departments or employees) query the outsourced databases for patient or student information. We call these entities *database users*. And we simply call servers in the cloud, which store the database, *cloud server*.

On one hand, due to its benefits, companies are excited by the public debut of SAAS. On the other hand, companies are reticent about adopting SAAS. One of the major concerns is the privacy. In case of the health care provider scenario, a personal medical record contains one's medical history, details about one's lifestyle (e.g. smoking), and family medical history. It may also include one's laboratory test results, medication prescribed and reports indicating the result of operations. Therefore a personal medical record is considered by a patient to be highly sensitive and should be kept in confidentiality as per public law [3]. In case of educational institution scenario, a student's record which includes academic transcripts, general counseling information and financial situation such as loan collection records is considered to be highly sensitive by students and therefore required to be kept confidential according to federal law [2]. Moreover, for the above two examples, database owners should have access control over their database. For example, a diabetes department physician should not access cosmetic surgery department's patients records without authorization. A biological department staff should not access arts department students' record without approval. Outsourcing data may mean database owner loses its confidentiality control over data and access control over users.

Now we start to clarify different types of privacy challenges during the deployment of cloud service. From the perspective of the database owner, three challenges arise.

- *Challenge 1:* how to protect outsourced data from theft by hackers or malware infiltrating the cloud server? Encryption by the cloud server and authenticated access by users seem to be a straightforward solution. However, careful consideration should be given to both encryption method and its granularity.

- *Challenge 2:* how to protect outsourced data from abuse by the cloud server? A trivial solution is for the owner to encrypt the database prior to outsourcing. Subsequently, users (armed with the decryption key(s)) can download the entire encrypted database, decrypt it and perform querying *in situ*. Clearly, this negates most benefits of using the cloud. A more elegant approach is to use searchable encryption. Unfortunately, current searchable encryption techniques only support simple search (attribute=value), as opposed to complicated SQL, queries.

- *Challenge 3:* how to realize content-level fine-grained access control for users? This challenge is even harder to solve as it requires variable decryption capabilities for different users. Even trivial solution to the second challenge does not solve this challenge as it gives each user equal decryption capability (same decryption key). An ideal solution would entail the database owner issuing a given user a key that only allows the user to search and decrypt certain records.

From user's perspective, three more challenges arise.

- *Challenge 4:* how to query the cloud server without revealing query details? Learning user's query details means learning user's possibly sensitive search interest. In addition, by learning user queries, the cloud server gradually learns the information in the encrypted database.

- *Challenge 5:* how to hide query contents (e.g., values used in "attribute=value" queries) from the database owner? For the database owner to exercise access control over its outsourced data, a user should first obtain an approval from the database owner over its query contents. However, in some cases, the user may want to get the approval without revealing its query contents even to the database owner. This is the case when the user happens to be a high-level executive who is automatically qualified to search any value and is not willing to reveal query to anyone.

- *Challenge 6:* how to hide query contents while assuring database owner the hidden contents are authorized by some certificate authority (CA)? Such challenge happens, for example, when the user is FBI who does not want to reveal the person it is investigating while database owner wants to get some confidence by making sure FBI is authorized by the court to do this investigation.

To address the above challenges, we need a scheme for the scenario shown in Fig. 1. In the initial deployment phase, the owner encrypts its database and transfers it to the cloud server. The encryption scheme should guarantee that no plaintext is leaked in the encrypted database, thereby addressing challenges 1–2. When a user poses an SQL query, such as:

"select from sample where ((last_name='Lobb' AND birth_date='3/26/1983') OR blood_type='B')"

it first obtains a search token and decryption key from the database owner. Then, the user supplies the search token to the cloud server who uses the token to search the encrypted database. Matching encrypted records are returned to the user who finally decrypts them. The search token and the decryption key should only allow the user to search and decrypt records meeting the conditional expression in the specific query, therefore addressing challenge 3. The search token should not reveal the conditional expression specified by the user, therefore solving challenge 4. Further, the user should be able to get the search token and decryption key without letting database owner know the query contents in order to solve challenge 5. Finally, to solve challenge 6, database owner, even though not knowing the query contents, should be able to verify if these contents are authorized by a CA.

This paper extends its previous conference version [23] by including detailed security proofs. In this paper, we present a new scheme that addresses aforementioned requirements. It relies on attribute-based encryption [20] and blind Boneh-Boyen weak signature scheme [5]. In fact, we amend the standard attribute-based encryption to make it privately searchable in the cloud computing scenario. Furthermore, we use the blind Boneh-Boyen signature scheme to let a user obliviously retrieve a search token and decryption key. Moreover, blind search token and decryption key extraction procedure can be coupled with CA authorization on user's input.

This paper aims to make four contributions: First, we define the adversary and security model for an encryption scheme aimed at the cloud database system. Second, we construct an encryption scheme that protects data privacy and allows access control. Third, we develop techniques for a user to retrieve search token and decryption key from database owner without revealing query contents. Fourth, we make it possible that the database owner, without knowing query contents, can make sure these contents are authorized by CA.

The rest of the paper is organized as follows. Sec. 2 overviews related work. Next, Sec. 3 defines the function and security model. Then, Sec. 4 discusses some background issues. The new scheme is presented in Sec. 5, followed by Sec. 6 that analyzes its performance. An in-depth performance evaluation is shown in Sec. 7. Limitations of our scheme are discussed in Sec. 8. Finally, Sec. 9 concludes this paper. A complete security proof is shown in Appx A.

2 Related Work

Private Information Retrieval and Oblivious Transfer: Private Information Retrieval (PIR) [17] allows a user to retrieve an item from a server's (public) database without the latter learning which item is being retrieved. While PIR is not concerned with privacy of the server database, Oblivious Transfer (OT) [18] adds an additional requirement that the user should not receive records beyond those requested. Olumofin and Goldberg [26] apply PIR/OT concepts to relational databases in order to hide user SQL queries from the database server.

There are significant differences between these approaches and our work. First, these approaches target a user/server scenario and it is unclear how to extend them to the cloud setting with the additional requirement of protecting data from untrusted cloud server. Second, a user can query any items inside the database and there is no way to enforce access control in these approaches.

Search on encrypted database: Searching on encrypted data (SoE), also known as privacy preserving keyword-based retrieval over encrypted data, was introduced in the symmetric key setting by Song, et al. [33]. This scheme allows a user to store its symmetrically encrypted data on an untrusted server and later search for a specific keyword by giving the server a search capability, that does not reveal the keyword or any plaintext. Its security and efficiency was later improved in [14] and [35]. Golle, et al. [19] developed a symmetric-key version of SoE that supports conjunctive keyword search. Boneh, et al. [9] later proposed a public-key version of encryption with keyword search (PEKS), where any party in possession of the public key can encrypt and send encryption to an untrusted server, while only the owner of the corresponding private key can generate keyword search capabilities. The server can identify all messages containing the searching keyword, but learning nothing else.

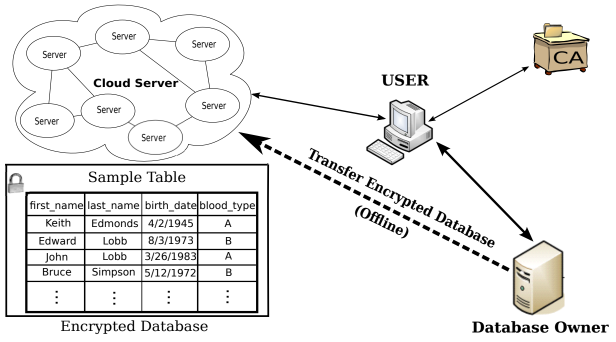
Our work is different from SoE and PEKS since it supports flexible access control (any monotonic access structure) on encrypted data, i.e. the database owner can issue a user a decryption key that only decrypts data meeting a certain conditional expression. Also, our scheme supports oblivious (search token/decryption key) retrieval.

Attribute-based encryption: Sahai and Waters [29] introduced the concept of Attribute-Based Encryption (ABE) where a user's keys and ciphertexts are labeled with sets of descriptive attributes and a particular key can decrypt a particular ciphertext only if the cardinality of the intersection of their labeled attributes exceeds a certain threshold. Later, Goyal, et al. [20] developed a Key-Policy Attribute-Based Encryption (KP-ABE) where the trusted authority (master key owner) can generate user private keys associated with any monotonic access structures consisting of **AND, OR** or threshold gates. Only ciphertexts that satisfy the private key's access structure can be decrypted. Bethencourt, et al. [6] explore the concept of Ciphertext-Policy Attribute-Based Encryption where each ciphertext is associated with an access structure that specifies which type of secret keys can decrypt it. Ostrovsky, et al. [27] extended [20] by allowing negative constraints in a key access structure.

Our scheme is derived from that in [20]. However, compared to traditional ABE, there are several notable differences. First, ABE only achieves payload hiding, i.e., attributes are revealed in plaintext, while our scheme hides the attributes. Second, ABE does not support private search on encrypted data, while our scheme does. Third, ABE does not support oblivious private key retrieval from the authority, while our scheme does.

Predicate encryption: Predicate encryption can be considered as attribute-based encryption supporting attribute-hiding. Ciphertexts are associated with a set of hidden attributes I. The master secret key owner has the fine-grained control over access to encrypted data by generating a secret key sk_f corresponding to predicate f; sk_f can be used to decrypt a ciphertext associated with attribute I if and only if $f(I) = 1$.

Several results have yielded predicate encryption schemes for different predicates. Waters, et al. constructed an equality tests predicate encryption scheme [34]. Shi and Waters [32] constructed a conjunction predicate encryption scheme. In [31], Shi, et al. proposed a scheme for range queries. Boneh and Waters [10] developed a scheme that handles conjunctions and range queries while satisfying a stronger notion of attribute hiding. Katz, et al. [22] moved a step further by making predicate encryption support inner products, therefore supporting disjunction and polynomial evaluation. Shi, et al. [30] noticed that public key predicate encryption may inherently reveal the query predicate inside a token and proposed a



first_name	last_name	birth_date	blood_type
Keith	Edmonds	4/2/1945	A
Edward	Lobb	8/3/1973	B
John	Lobb	3/26/1983	A
Bruce	Simpson	5/12/1972	B
⋮	⋮	⋮	⋮

Figure 1: Cloud storage architecture.

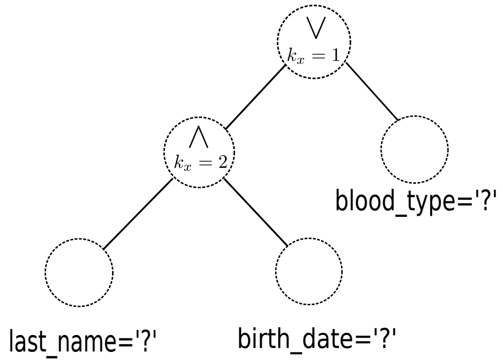


Figure 2: Access tree example

symmetric key version that supports inner product that addresses this problem.

Our approach is different in several respects. First, no concrete private search scheme exists in predicate encryption. Although a predicate-only version is enough for private search [22], requiring private search on a cloud server and access control for users probably means that two separate implementations of predicate encryption are needed. Second, our scheme supports more flexible access control; although, range queries are not covered. Finally, no oblivious retrieval of decryption key for predicate encryption exists so far.

3 Problem Definition

3.1 Problem Description

Fig. 1 shows the architecture of the envisaged cloud storage scenario. There are four entities: the cloud server ($\mathcal{S}$), the database owner ($\mathcal{DO}$), the database user ($\mathcal{U}$) and the certificate authority ($\mathcal{CA}$). $\mathcal{DO}$'s database table consists of w attributes $\{\alpha_1, \alpha_2, \ldots, \alpha_w\}$. Let $\Omega = \{1, \cdots, w\}$. For ease of description, we assume that every attribute is searchable. Each record m includes w values: $\{v_i\}_{1 \leq i \leq w}$ with each v_i corresponding to attribute α_i. Fig. 1 also illustrates a sample database. The first row describes attribute names and each subsequent row denotes a record.

$\mathcal{U}$ may issue $\mathcal{S}$ any SQL query with monotonic access structure. By monotonic access structure, we mean a boolean formula only involving 'AND/OR' combinations. We use an **access tree** (see Sec. 4.2 for details) to describe any monotonic access structure. In our context, the access tree describes a combination of 'AND/OR' of attribute names, without specifying their values. For example, Fig. 2 depicts one type of access tree corresponding to a conditional expression ((last_name=? AND birth_date=?) OR blood_type=?). If concrete values are supplied together with an access tree, a complete conditional expression can be defined. For example, if a value set (Lobb, 3/26/1983, B) is specified, the expression will be ((last_name='Lobb' AND birth_date='3/26/1983') OR blood_type='B'). We use $\mathcal{T}_\gamma$ to denote an access tree constructed over a subset γ of Ω and use $\mathbf{v}_\gamma$ to describe a set of values for $\mathcal{T}_\gamma$ to completely define a conditional expression. A complete record can be viewed as $\mathbf{v}_\Omega$. We use $\mathcal{T}_\gamma(\mathbf{v}_\gamma, \mathbf{v}_{\gamma'})$ to test whether a set of values $\mathbf{v}_{\gamma'}$ satisfies the conditional expression defined by $\mathcal{T}_\gamma$ and $\mathbf{v}_\gamma$.

Our basic encryption scheme is a set of components: **Setup, Encrypt, Extract, Test, Decrypt**. Before starting, $\mathcal{DO}$ runs **Setup** to initialize some parameters. Then $\mathcal{DO}$ runs **Encrypt** over each record in its table to form an encrypted database. The encrypted database is exported to $\mathcal{S}$ (off-line) and $\mathcal{DO}$ can insert new encrypted items later. Whenever $\mathcal{U}$ forms an SQL query, it runs **Extract** with $\mathcal{DO}$ to extract a search token and decryption key. Then, $\mathcal{U}$ hands the search token to $\mathcal{S}$ and the latter runs **Test** over each encrypted record, in order to find matching records. After that, $\mathcal{S}$ sends matching records back and $\mathcal{U}$ runs **Decrypt** to recover plaintext records. If additional requirement that $\mathcal{DO}$ learns nothing about query content is needed, $\mathcal{U}$ can run **BlindExtract** instead of **Extract** with $\mathcal{DO}$. If further requirement that $\mathcal{U}$'s query should be **Authorize**d by CA is needed, $\mathcal{U}$ can engage in **AuthorizedBlindExtract** with $\mathcal{DO}$. We define each function in more detail below.

3.2 Basic Scheme Definition

The basic scheme includes following components:

Setup(1^k): on input a security parameter 1^k, $\mathcal{DO}$ outputs parameters $params$, $\mathcal{DO}$'s master key $msk_{\mathcal{DO}}$.

Encrypt$(\mathcal{DO}(params, msk_{\mathcal{DO}}, \mathbf{v}_\Omega))$: $\mathcal{DO}$ on input $params, msk_{\mathcal{DO}}$ and a record $\mathbf{v}_\Omega$, outputs a ciphertext.

Extract$(\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{DO}(params, msk_{\mathcal{DO}}))$: $\mathcal{U}$ on input $(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ and $\mathcal{DO}$ on input $(params, msk_{\mathcal{DO}})$ engage in an interactive protocol. At the end, $\mathcal{U}$ outputs a search token $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and a decryption key $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$, and $\mathcal{DO}$ outputs $(\mathcal{T}_\gamma, \mathbf{v}_\gamma)$.

Test$(\mathcal{S}(params, tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}, C))$: $\mathcal{S}$ on input parameters $params$, a search token $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and a ciphertext $C = \textbf{Encrypt}(msk_{\mathcal{DO}}, \mathbf{v}'_\Omega)$, outputs "yes" if $\mathcal{T}_\gamma(\mathbf{v}_\gamma, \mathbf{v}'_\Omega) = 1$ and "no" otherwise.

Decrypt$(\mathcal{U}(params, tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)} sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}, C))$: $\mathcal{U}$ on input $params$, a search token $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$, a decryption key $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and a ciphertext $C = \textbf{Encrypt}(msk_{\mathcal{DO}}, \mathbf{v}'_\Omega)$, outputs $\mathbf{v}'_\Omega$ if $\mathcal{T}_\gamma(\mathbf{v}_\gamma, \mathbf{v}'_\Omega) = 1$ and $\perp$ otherwise.

3.3 Blind Extraction Definition

In order to protect $\mathcal{U}$'s query from $\mathcal{DO}$, we need to replace **Extract** with a blinded version, called **BlindExtract**.

BlindExtract$(\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{DO}(params, msk_{\mathcal{DO}}))$: $\mathcal{U}$ on input $(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ and $\mathcal{DO}$ on input $(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma)$ engage in an interactive protocol. $\mathcal{U}$'s output is a search token $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and a decryption key $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$, and $\mathcal{DO}$'s output is $\mathcal{T}_\gamma$.

Sometimes, it makes more sense to require $\mathcal{U}$ to prove that its input in **BlindExtract** is authorized by a CA before $\mathcal{U}$ can get anything useful. In order to realize that, we introduce two other functions **Authorize** and **AuthorizedBlindExtract**. **Authorize** helps a $\mathcal{U}$ get a commitment ψ and a signature σ from a CA. In **AuthorizedBlindExtract**, $\mathcal{DO}$ is provided with $\mathcal{T}_\gamma, \psi, \sigma$ while $\mathcal{U}$ can prove statements about commitment ψ using zero-knowledge proof.

Authorize$(\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{CA}(params, msk_{\mathcal{CA}}))$: $\mathcal{CA}$ generates a commitment ψ over $\mathcal{U}$'s input $(\mathcal{T}_\gamma, \mathbf{v}_\gamma)$, the randomness $open$ used to compute ψ and a signature σ over ψ. $\mathcal{CA}$'s output is $(\mathcal{T}_\gamma, \mathbf{v}_\gamma, \psi, open, \sigma)$. $\mathcal{U}$'s output is $(\psi, open, \sigma)$.

AuthorizedBlindExtract$(\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma, \psi, open, \sigma), \mathcal{DO}(params, msk_{\mathcal{DO}}))$: $\mathcal{U}$ on input $(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma, \psi, open, \sigma)$ and $\mathcal{DO}$ on input $(params, msk_{\mathcal{DO}})$ engage in an interactive protocol. $\mathcal{DO}$'s output is $(\mathcal{T}_\gamma, \psi, \sigma)$. If $\psi = \textbf{Commit}((\mathcal{T}_\gamma, \mathbf{v}_\gamma), open)$ and $\mathsf{Vrfy}_{pk_{\mathcal{CA}}}(\psi, \sigma) = 1$, $\mathcal{U}$'s output is a search token $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and a decryption key $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$, and otherwise, $\mathcal{U}$ outputs $\perp$.

3.4 Adversary Model and Security Requirement

In this paper, we assume the malicious adversary model (as opposed to semi-honest, a.k.a "honest-but-curious") . A malicious adversary can arbitrarily deviate from the prescribed protocols. We also assume that $\mathcal{U}$ may collude with $\mathcal{S}$. However, $\mathcal{DO}$ does not collude with any party. In Appx. A, we will prove our scheme is secure against malicious adversary according to Def. 1, 2 and 3.

For the basic scheme, we define adversary's advantage by defining a security game under chosen plaintext attack in a selective set model, similar to [20].

Definition 1. (Selective-Set Secure (IND-SS-CPA)). *Let k be a security parameter. Above scheme is IND-SS-CPA-secure if every p.p.t. adversary $\mathcal{A}$ has an advantage negligible in k for the following game: (1) Run* **Setup***(1^k) to obtain $(params, msk_{\mathcal{DO}})$, and give params to $\mathcal{A}$. (2) $\mathcal{A}$ outputs two records m_1, m_2 to be challenged on (3) $\mathcal{A}$ may query an oracle $\mathcal{O}_{\textbf{Extract}}(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ such that $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m_1) \neq 1$ and $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m_2) \neq 1$. (4) Select a random bit b and give $\mathcal{A}$ the challenge $c^* \leftarrow$* **Encrypt***$(params, msk_{\mathcal{DO}}, m_b)$. (5) $\mathcal{A}$ may continue to query oracle $\mathcal{O}_{\textbf{Extract}}(\cdot)$ under the same conditions as before. (6) $\mathcal{A}$ outputs a bit b'. We define $\mathcal{A}$'s advantage in the above game as $|\Pr[b' = b] - 1/2|$.*

BlindExtract must satisfy two security properties: *Leak-free Extract* [21] and *Selective-failure Blindness* [12]. Informally, the former means that a malicious $\mathcal{U}$ cannot learn more by executing the **BlindExtract** with an honest $\mathcal{DO}$ than by executing **Extract** with an honest $\mathcal{DO}$. Whereas, *Selective-failure Blindness* means that a malicious $\mathcal{DO}$ cannot learn anything about $\mathcal{U}$'s choice of $\mathbf{v}_\gamma$ during **BlindExtract**. Moreover, $\mathcal{DO}$ cannot cause **BlindExtract** to fail based on $\mathcal{U}$'s choice. Now we formally define *Leak-free Extract* and *Selective-failure Blindness*:

Definition 2. (Leak-Free Extract). **BlindExtract** *protocol is leak free if, for all p.p.t. adversaries $\mathcal{A}$, there exists an efficient simulator such that for every value k, $\mathcal{A}$ cannot determine whether it is playing Game Real or Game Ideal with non-negligible advantage, where*

Game Real: Run **Setup***(1^k). As many times as $\mathcal{A}$ wants, $\mathcal{A}$ chooses its $\mathcal{T}_\gamma, \mathbf{v}_\gamma$ and executes* **BlindExtract***$(\cdot)$ with $\mathcal{DO}$.*

Game Ideal: Run **Setup***(1^k). As many times as $\mathcal{A}$ wants, $\mathcal{A}$ chooses its $\mathcal{T}_\gamma, \mathbf{v}_\gamma$ and executes* **BlindExtract***$(\cdot)$ with a simulator which does not know $msk_{\mathcal{DO}}$ and only queries a trusted party to obtain $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$ and $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$.*

Definition 3. (Selective-Failure Blindness). **BlindExtract** *is selective-failure blind if every p.p.t. adversary $\mathcal{A}$ has a negligible advantage in the following game: First, $\mathcal{A}$ outputs params and a pair of $(\mathcal{T}, \mathbf{v}_1)$, $(\mathcal{T}, \mathbf{v}_2)$. A random bit b is chosen. $\mathcal{A}$ is given black-box access to two oracles $\mathcal{U}(params, \mathcal{T}, \mathbf{v}_b)$ and*
$\mathcal{U}(params, \mathcal{T}, \mathbf{v}_{1-b})$. The $\mathcal{U}$ algorithm produces local output $s_b = (tk_{(\mathcal{T}, \mathbf{v}_b)}, sk_{(\mathcal{T}, \mathbf{v}_b)})$ and $s_{1-b} = (tk_{(\mathcal{T}, \mathbf{v}_{1-b})}, sk_{(\mathcal{T}, \mathbf{v}_{1-b})})$ respectively. If $s_b \neq \perp$ and $s_{1-b} \neq \perp$ then $\mathcal{A}$ receives (s_0, s_1). If $s_b = \perp$ and $s_{1-b} \neq \perp$ then $\mathcal{A}$ receives $(\perp, \varepsilon)$. If $s_b \neq \perp$ and $s_{1-b} = \perp$, then $\mathcal{A}$ receives $(\varepsilon, \perp)$. If $s_b = \perp$ and $s_{1-b} = \perp$, then $\mathcal{A}$ receives $(\perp, \perp)$. Finally, $\mathcal{A}$ outputs its guess bit b'. We define $\mathcal{A}$'s advantage in the above game as $|\Pr[b' = b] - 1/2|$.

4 Preliminaries

4.1 Notation

Let $\{0,1\}^l$ denote the set of integers of maximum length l, i.e. the set $[0, 2^l - 1]$ of integers. We employ the security parameters $l_\phi, l_{\mathcal{H}}$ where l_ϕ (80) is the security parameter controlling the statistical zero-knowledge property, $l_{\mathcal{H}}$ (160) is the output length of the hash function used for the Fiat-Shamir heuristic. $\mathcal{H}(\cdot)$ and $\mathcal{H}'(\cdot)$ denote two distinct hash function. We use Enc_{pk}^{hom} and Dec_{sk}^{hom} to denote homomorphic encryption and decryption (respectively) under public key pk (or secret key sk). We use Enc_k^{sym} and Dec_k^{sym} to denote symmetric encryption and decryption under key k. We define Lagrange Coefficient as $\Delta_{i,S} = \prod_{j \in S, j \neq i} \frac{j}{j-i}$. Let Ω denote attributes index set, i.e. $\Omega = \{1, \cdots, w\}$. $\mathcal{DO}$'s private and public keys are $sk_{\mathcal{DO}}$ and $pk_{\mathcal{DO}}$, respectively. *server*'s master key is $msk_{\mathcal{DO}}$. $\mathcal{CA}$'s private and public keys are $sk_{\mathcal{CA}}$ and $pk_{\mathcal{CA}}$.

4.2 Access Tree

We use $\mathcal{T}$ to denote a tree representing an access structure. $\mathcal{T}$ represents a combination of 'AND/OR' of attribute names without specifying their values, as shown in Fig. 2. An access structure $\mathcal{T}_\gamma$ defined over a set γ of attributes, coupled with a set of values $\mathbf{v}_\gamma$ defined over the same set, completely defines a conditional expression (See Sec. 3.1 for example). We use $\mathcal{T}_\gamma(\mathbf{v}_\gamma, \mathbf{v})$ to test whether another set of values $\mathbf{v}$ satisfies the condition defined by $\mathcal{T}_\gamma$ and $\mathbf{v}_\gamma$. Each non-leaf node represents a threshold gate, described by its children and a threshold value. Let num_x be the number of children of a node x. The threshold value associated with node x is denoted by k_x that is either 1 or num_x, depending on the threshold gate. In case of an OR gate, $k_x = 1$; in case of an AND gate, $k_x = num_x$. Each leaf node x is described by an attribute with a threshold $k_x = 1$. Standard tree data structures can be used to represent and store $\mathcal{T}$. Since $\mathcal{T}_\gamma$ is exposed to $\mathcal{S}$ in **Test**, to prevent $\mathcal{S}$ from learning database schema, each leaf node can store an attribute index instead of the attribute name.

To facilitate working with the access trees, we define a few functions. We denote the parent of the node x as $parent(x)$. $node(\alpha_i)$ returns the leaf node corresponding to attribute α_i. $attr(x)$ is defined only if x is a leaf node; it returns the attribute index i of α_i associated with x. Access tree $\mathcal{T}$ also defines an ordering between the children of every node, i.e. each child y of a node x are numbered from 1 to num_x. $index(y)$ returns this number associated with the node y. Let S_x denote a set $[1, \ldots, num_x]$. Finally, let $child_i(x)$ return the ith child of node x.

We also define $\Gamma_{\mathcal{T}_\gamma}$ as a set of minimum subsets of γ that satisfies $\mathcal{T}_\gamma$. By "minimum", we mean the subset cannot become smaller while still satisfying $\mathcal{T}_\gamma$. For example, in Fig. 2, $\Gamma_{\mathcal{T}_\gamma} = \{\{1,2\},\{3\}\}$ where $1,2,3$ is the index of attribute $last\_name, birth\_date, blood\_type$ respectively. Here $\Gamma_{\mathcal{T}_\gamma}$ means that either $\{last\_name, birth\_date\}$ or $\{blood\_type\}$ can satisfy $\mathcal{T}_\gamma$. We can determine $\Gamma_{\mathcal{T}_\gamma}$ in a down-top manner. For each leaf node, define $\mathbb{S}_x = \{attr(x)\}$. For any other node x, $\mathbb{S}_x = \cup_{i \in S_x} \mathbb{S}_{child_i(x)}$ if $k_x = 1$. Otherwise if $k_x > 1$, $\mathbb{S}_x = \{x' : x' = \cup_{1 \le i \le k_x} x'_i, \forall x'_i \in \mathbb{S}_{child_i(x)}\}$. And the resulting $\mathbb{S}_r$ at root node r is $\Gamma_{\mathcal{T}_\gamma}$. For $\gamma' \in \Gamma_{\mathcal{T}_\gamma}$, we define $\mathcal{T}_{\gamma'}$ as a subgraph of $\mathcal{T}_\gamma$ with only attributes in γ' as leaves. For example, in Fig. 2, if $\gamma' = \{1,2\}$, then $\mathcal{T}_{\gamma'}$ would be the left-hand subtree of the root node. Note in $\mathcal{T}_{\gamma'}$ each non-leaf node x's k_x should be its number of children, i.e., a conjunctive gate, since γ' is a minimum satisfiable subset.

4.3 Homomorphic Encryption

Here, we elect to use the well-known additively homomorphic public key encryption scheme–Paillier encryption [28] which is easy to implement and amend for proofs of knowledge. Let n denote an RSA modulus, $\mathsf{h} = \mathsf{n} + 1$ and g be an element of order $\phi(\mathsf{n}) \bmod \mathsf{n}^2$. Let $sk = \{\phi(\mathsf{n})\}$ and $pk = \{\mathsf{g}, \mathsf{n}\}$. Encryption is defined as $c = \mathsf{Enc}^{hom}_{pk}(m) = \mathsf{h}^m \mathsf{g}^r \bmod \mathsf{n}^2$ where $r \in_R \mathbb{Z}_{\phi(\mathsf{n})}$. Corresponding decryption is defined as: $\mathsf{Dec}^{hom}_{sk}(c) = \left[\frac{(c^{\phi(\mathsf{n})} \bmod \mathsf{n}^2) - 1}{\mathsf{n}} \cdot \phi(\mathsf{n})^{-1} \bmod N\right]$. Note that, to encrypt, we use $\mathsf{h}^m \mathsf{g}^r$ instead of standard $\mathsf{h}^m r^{\mathsf{n}}$. If the order of g has no factor of n and is greater than 2, g^r is a random element from the same subgroup as r^{n}. Therefore $\mathsf{h}^m \mathsf{g}^r$ has the same distribution as $\mathsf{h}^m r^{\mathsf{n}}$. The purpose of using the former is to facilitate zero-knowledge proofs.

4.4 Zero-Knowledge Proof

Our scheme uses various protocols to prove knowledge of, and relations among, discrete logarithms. To describe these protocols, we use the notation introduced by Camenisch and Stadler [13]. For instance, $PK\{(a,b,c) : y = g^a h^b \wedge \mathfrak{y} = \mathfrak{g}^a \mathfrak{h}^c \wedge s \le b \le t\}$ denotes a zero-knowledge proof of knowledge of integers a,b,c such that $y = g^a h^b$ and $\mathfrak{y} = \mathfrak{g}^a \mathfrak{h}^c$ holds and $s \le b \le t$. The convention is that everything inside parentheses is only known to the prover, while all other parameters are known to both prover and verifier.

The technique for a proof of knowledge of a representation of an element $y \in G$ with respect to several bases $z_1, \dots, z_v \in G$, i.e., $PK\{(a_1, \cdots, a_v) : y = z_1^{a_1} \cdots z_v^{a_v}\}$, is presented in [16]. A proof of equality of discrete logarithms of two group elements $y_1, y_2 \in G$ to bases $g \in G$ and $h \in G$, respectively, i.e., $PK\{(a) : y_1 = g^a \wedge y_2 = h^a\}$, is given in [15]. Generalizations to proving equalities among representations of elements $y_1, \dots, y_v \in G$ to bases $g_1, \dots, g_v \in G$ are straightforward [13]. Boudot [11] demonstrates proof of knowledge of a discrete logarithm of $y \in G$ with respect to $g \in G$ such that $\log_g y$ lies in integer interval $[s,t]$, i.e., $PK\{(a) : y = g^a \wedge a \in [s,t]\}$ under the strong RSA assumption and the assumption that the prover does not know the factorization of the RSA modulus.

4.5 Bilinear map

We now review some general notions about efficiently computable bilinear maps.

Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be two multiplicative cyclic groups of prime order q. Let g be a generator of $\mathbb{G}_1$ and $\hat{e}$ be a bilinear map, $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$. The bilinear map $\hat{e}$ has the following properties:

1. Bilinearity: for all $u, v \in \mathbb{G}_1$ and $a, b \in \mathbb{Z}_p$, we have $\hat{e}(u^a, v^b) = \hat{e}(u,v)^{ab}$
2. Non-degeneracy: $\hat{e}(g,g) \neq 1$.

We say that $\mathbb{G}_1$ is a bilinear group if the group operation in $\mathbb{G}_1$ and the bilinear map $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ are both efficiently computable.

4.6 Cryptographic Assumption

Our scheme's security is based on the decisional bilinear Diffie-Hellman (BDH) assumption [7] and Boneh-Boyen Hidden Strong Diffie-Hellman (BB-HSDH) assumption [8].

Assumption 1. (Decisional Bilinear Diffie-Hellman (BDH) assumption.) *Let $a, b, c, z \in \mathbb{Z}_q$ be chosen at random and g be a generator of $\mathbb{G}_1$. We say that the BDH problem is hard if for all p.p.t. adversaries $\mathcal{A}$ there exists a negligible function* negl *such that $|Pr[\mathcal{A}(g^a, g^b, g^c, \hat{e}(g,g)^{abc}) = 1] - Pr[\mathcal{A}(g^a, g^b, g^c, \hat{e}(g,g)^z) = 1]| \leq \mathsf{negl}(n)$ where in each case the probabilities are taken over the random choice of the generator g, the random choice of a, b, c, z in $\mathbb{Z}_q$ and the random bits consumed by $\mathcal{A}$.*

Assumption 2. (Boneh-Boyen Hidden Strong Diffie-Hellman (BB-HSDH)). *Let $x, c_1, \cdots c_t \in_R \mathbb{Z}_q$. On input $g, g^x, u \in \mathbb{G}_1, h, h^x \in \mathbb{G}_2$ and the tuple $\{g^{1/(x+c_l)}, c_l\}_{l=1\ldots t}$, it is computationally infeasible to output a new tuple $(g^{1/(x+c)}, h^c, u^c)$.*

5 Scheme

We present our scheme Π which consists of following algorithms. The full security proof is given in Appx A.

Setup(1^k): Run $\mathcal{G}(1^k)$ to obtain $(q, \mathbb{G}_1, \mathbb{G}_2, \hat{e}, \mathsf{n}, \mathsf{g}, \mathfrak{n}, \mathfrak{g}, \mathfrak{h})$. n is an RSA modulus larger than $2^k q^2$ with generator g. Let $sk_{\mathcal{DO}} = \phi(\mathsf{n})$ and $pk_{\mathcal{DO}} = \{\mathsf{g}, \mathsf{n}\}$. In other words, only $\mathcal{DO}$ knows the factors of n. $\mathfrak{n}$ is another RSA modulus with generator $\mathfrak{g}$ and $\mathfrak{h}$. Note neither factors of n nor $\log_{\mathfrak{g}} \mathfrak{h}$ is known to any party. Pick secret parameters t, t', y, y' which are only known to $\mathcal{DO}$. Make $Y = \hat{e}(g,g)^y$, $Y' = \hat{e}(g,g)^{y'}$, $T = g^t$, $T' = g^{t'}$, $e_t = \mathsf{Enc}^{hom}_{pk_{\mathcal{DO}}}(t)$, $e_{t'} = \mathsf{Enc}^{hom}_{pk_{\mathcal{DO}}}(t')$, and π^s proving e_t and $e_{t'}$ are well formed. Output $params \leftarrow (Y, Y', T, T', e_t, e_{t'}, \pi^s, pk_{\mathcal{DO}}, pk_{\mathcal{CA}}, \mathfrak{n}, \mathfrak{g}, \mathfrak{h})$, $msk_{\mathcal{DO}} \leftarrow (t, t', y, y', sk_{\mathcal{DO}})$.

1. **Setup** algorithm picks $r, r' \in_R \mathbb{Z}_{\phi_{\mathsf{n}}}$, computes $e_t = \mathsf{h}^t \mathsf{g}^r$, $e_{t'} = \mathsf{h}^{t'} \mathsf{g}^{r'}$ and

$$\pi^s = PK\left\{ \left(t, t', r, r', \mathfrak{r}, \mathfrak{r}' \right) : \begin{array}{l} e_t = \mathsf{h}^t \mathsf{g}^r \bmod \mathsf{n}^2 \wedge c_t = \mathfrak{g}^t \mathfrak{h}^{\mathfrak{r}} \bmod \mathfrak{n} \wedge t \in \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}} + 2} \wedge \\ e_{t'} = \mathsf{h}^{t'} \mathsf{g}^{r'} \bmod \mathsf{n}^2 \wedge c_{t'} = \mathfrak{g}^{t'} \mathfrak{h}^{\mathfrak{r}'} \bmod \mathfrak{n} \wedge t' \in \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}} + 2} \end{array} \right\}$$

which is instantiated as follows:

   (a) Pick random $r_t, r_{t'} \in_R \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}}}$, $r_r, r_{r'} \in_R \mathbb{Z}_{\phi(\mathsf{n})}$, $r_{\mathfrak{r}}, r_{\mathfrak{r}'} \in_R \{0,1\}^{l_{\mathfrak{r}} + l_\phi + l_{\mathcal{H}}}$ and compute $\tilde{e}_t = \mathsf{h}^{r_t} \mathsf{g}^{r_r} \bmod \mathsf{n}^2$, $\tilde{c}_t = \mathfrak{g}^{r_t} \mathfrak{h}^{r_{\mathfrak{r}}} \bmod \mathfrak{n}$, $\tilde{e}_{t'} = \mathsf{h}^{r_{t'}} \mathsf{g}^{r_{r'}} \bmod \mathsf{n}^2$, $\tilde{c}_{t'} = \mathfrak{g}^{r_{t'}} \mathfrak{h}^{r_{\mathfrak{r}'}} \bmod \mathfrak{n}$

   (b) Compute $c = \mathcal{H}(\mathsf{g}||\mathsf{h}||\mathfrak{g}||\mathfrak{h}||e_t||c_t||\tilde{e}_t||\tilde{c}_t||e_{t'}||c_{t'}||\tilde{e}_{t'}||\tilde{c}_{t'})$.

   (c) Make $\pi^s = (c, s_t, s_r, s_{\mathfrak{r}}, s_{t'}, s_{r'}, s_{\mathfrak{r}'})$ where $s_t = r_t + c \cdot t$, $s_r = r_r + c \cdot r$, $s_{\mathfrak{r}} = r_{\mathfrak{r}} + c \cdot \mathfrak{r}$, $s_{t'} = r_{t'} + c \cdot t'$, $s_{r'} = r_{r'} + c \cdot r'$, $s_{\mathfrak{r}'} = r_{\mathfrak{r}'} + c \cdot \mathfrak{r}'$.

   Setup publishes $e_t, e_{t'}, \pi^s$.

2. $\mathcal{U}$ verifies π^s by

   (a) computing $\hat{e}_t = e_t^{-c} \mathsf{h}^{s_t} \mathsf{g}^{s_r}$, $\hat{c}_t = c_t^{-c} \mathfrak{g}^{s_t} \mathfrak{h}^{s_{\mathfrak{r}}}$, $\hat{e}_{t'} = e_{t'}^{-c} \mathsf{h}^{s_{t'}} \mathsf{g}^{s_{r'}}$, $\hat{c}_{t'} = c_{t'}^{-c} \mathfrak{g}^{s_{t'}} \mathfrak{h}^{s_{\mathfrak{r}'}}$.

   (b) checking if $c \overset{?}{=} \mathcal{H}((\mathsf{g}||\mathsf{h}||\mathfrak{g}||\mathfrak{h}||e_t||c_t||\hat{e}_t||\hat{c}_t||e_{t'}||c_{t'}||\hat{e}_{t'}||\hat{c}_{t'})$

   (c) and checking if $s_t, s_{t'} \overset{?}{\in} \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}} + 1}$

   $\mathcal{U}$ receives $\{\{r_{v_i}, r'_{v_i}, c_{v_i}, c'_{v_i}\}_{i \in \gamma}, \sigma\}$ from CA.

   $\mathcal{U}$ chooses $r_{i,1}, r'_{i,1} \in_R \mathbb{Z}_q$, $r_{\mathcal{H}}, r_{\mathcal{H}'} \in_R \mathbb{Z}_{\phi(n)}$ and $r_{i,2}, r'_{i,2} \in_R \{0, 1, \ldots, 2^{l_q + l_\phi}\}$, computes $e_i = ((e_t \cdot \mathsf{h}^{\mathcal{H}(i,v_i)} \mathsf{g}^{r_{\mathcal{H}}})^{r_{i,1}}) \cdot \mathsf{h}^{r_{i,2} q} \mathsf{g}^{r_{q,i}} \bmod \mathsf{n}^2, e'_i = ((e_{t'} \cdot \mathsf{h}^{\mathcal{H}'(i,v_i)} \mathsf{g}^{r_{\mathcal{H}'}})^{r'_{i,1}}) \cdot \mathsf{h}^{r'_{i,2} q} \mathsf{g}^{r'_{q,i}} \bmod \mathsf{n}^2, \forall i \in \gamma$ and

$$\pi^c = PK\left\{ \left(\begin{array}{l} \{\mathcal{H}(i,v_i), \mathcal{H}'(i,v_i), \bar{\mathcal{H}}_i, \bar{\mathcal{H}}'_i\}_{i \in \gamma} \\ \{r_{i,1}, r'_{i,1}, r_{i,2}, r'_{i,2}\}_{i \in \gamma} \\ \{\bar{r}_i, \bar{r}'_i, r_{v_i}, u_i, u'_i\}_{i \in \gamma} \end{array} \right) : \begin{array}{l} \{e_i = e_t^{r_{i,1}} \cdot \mathsf{h}^{\bar{\mathcal{H}}_i} \cdot (\mathsf{h}^q)^{r_{i,2}} \cdot \mathsf{g}^{\bar{r}_i} \bmod \mathsf{n}^2 \\ \wedge e'_i = e_{t'}^{r'_{i,1}} \cdot \mathsf{h}^{\bar{\mathcal{H}}'_i} \cdot (\mathsf{h}^q)^{r'_{i,2}} \cdot \mathsf{g}^{\bar{r}'_i} \bmod \mathsf{n}^2 \\ \wedge c_{v_i} = \mathfrak{g}^{\mathcal{H}(i,v_i)} \mathfrak{h}^{r_{v_i}} \bmod \mathfrak{n} \wedge 1 = c_{v_i}^{r_{i,1}} \mathfrak{g}^{-\bar{\mathcal{H}}_i} \mathfrak{h}^{u_i} \bmod \mathfrak{n} \\ \wedge c'_{v_i} = \mathfrak{g}^{\mathcal{H}'(i,v_i)} \mathfrak{h}^{r'_{v_i}} \bmod \mathfrak{n} \wedge 1 = {c'_{v_i}}^{r'_{i,1}} \mathfrak{g}^{-\bar{\mathcal{H}}'_i} \mathfrak{h}^{u'_i} \bmod \mathfrak{n} \\ \wedge r_{i,1}, r'_{i,1} \in \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}} + 2} \\ \wedge r_{i,2}, r'_{i,2} \in \{0,1\}^{l_q + 2l_\phi + l_{\mathcal{H}} + 2}\}_{i \in \gamma} \end{array} \right\}$$

   which is instantiated as follows:

   (a) $\mathcal{U}$ picks random $\{r_{r_{i,1}}, r_{r'_{i,1}} \in_R \{0,1\}^{l_q + l_\phi + l_{\mathcal{H}}}, r_{r_{i,2}}, r_{r'_{i,2}} \in_R \{0,1\}^{l_q + 2l_\phi + l_{\mathcal{H}}}$, $r_{\bar{\mathcal{H}}_i}, r_{\bar{\mathcal{H}}'_i} \in_R \{0,1\}^{l_q + l_\phi + 2l_{\mathcal{H}}}, r_{\mathcal{H}(i,v_i)}, r_{\mathcal{H}'(i,v_i)} \in_R \{0,1\}^{l_\phi + 2l_{\mathcal{H}}}, r_{\bar{r}_i} \in_R \{0,1\}^{l_{\bar{r}_i} + l_\phi + l_{\mathcal{H}}}$ $r_{\bar{r}'_i} \in_R \{0,1\}^{l_{\bar{r}'_i} + l_\phi + l_{\mathcal{H}}}, r_{r_{v_i}}, r_{r'_{v_i}} \in_R \{0,1\}^{l_{r_{v_i}} + l_\phi + l_{\mathcal{H}}}, r_{u_i} \in_R \{0,1\}^{l_{u_i} + l_\phi + l_{\mathcal{H}}}, r_{u'_i} \in_R \{0,1\}^{l_{u'_i} + l_\phi + l_{\mathcal{H}}}\}_{i \in \gamma}$

   and computes $\{\tilde{e}_i = e_t^{r_{r_{i,1}}} \cdot \mathsf{h}^{r_{\bar{\mathcal{H}}_i}} \cdot (\mathsf{h}^q)^{r_{r_{i,2}}} \cdot \mathsf{g}^{r_{\bar{r}_i}} \bmod \mathsf{n}^2$, $\tilde{e}'_i = e_{t'}^{r_{r'_{i,1}}} \cdot \mathsf{h}^{r_{\bar{\mathcal{H}}'_i}} \cdot (\mathsf{h}^q)^{r_{r'_{i,2}}} \cdot \mathsf{g}^{r_{\bar{r}'_i}} \bmod \mathsf{n}^2$, $\tilde{c}_{v_i} = \mathfrak{g}^{r_{\mathcal{H}(i,v_i)}} \mathfrak{h}^{r_{r_{v_i}}} \bmod \mathfrak{n}$, $\tilde{c}'_{v_i} = \mathfrak{g}^{r_{\mathcal{H}'(i,v_i)}} \mathfrak{h}^{r_{r'_{v_i}}} \bmod \mathfrak{n}$, $\tilde{O}_{i,1} = c_{v_i}^{r_{r_{i,1}}} (1/\mathfrak{g})^{r_{\bar{\mathcal{H}}_i}} \mathfrak{h}^{r_{u_i}} \bmod \mathfrak{n}$, $\tilde{O}_{i,2} = {c'_{v_i}}^{r_{r'_{i,1}}} (1/\mathfrak{g})^{r_{\bar{\mathcal{H}}'_i}} \mathfrak{h}^{r_{u'_i}} \bmod \mathfrak{n}\}_{i \in \gamma}$

   (b) Compute $c = \mathcal{H}(\mathsf{g}||\mathsf{h}||\mathfrak{g}||\mathfrak{h}||\mathcal{V}(e_i)||\mathcal{V}(e'_i)||\mathcal{V}(c_{v_i})||\mathcal{V}(\tilde{e}_i)||\mathcal{V}(\tilde{e}'_i)||\mathcal{V}(\tilde{c}_{v_i})||\mathcal{V}(\tilde{c}'_{v_i})||\mathcal{V}(\tilde{O}_{i,1})||\mathcal{V}(\tilde{O}_{i,2}))$ where $\mathcal{V}(x_i) = x_{k_1}||\cdots||x_{k_j}||\cdots||x_{k_{|\gamma|}}$ in which $k_j \in \gamma$

Figure 3: The **AuthorizedBlindExtract** protocol

Encrypt($\mathcal{DO}(params, msk_{\mathcal{DO}}, m)$): To encrypt a record $m = \mathbf{v}_\Omega = \{v_1, \ldots, v_w\}$, $\mathcal{DO}$ chooses random

2. (c) Make $\pi^c = (c, \{s_{r_{i,1}}, s_{r'_{i,1}}, s_{r_{i,2}}, s_{r'_{i,2}}, s_{\bar{\mathcal{H}}_i}, s_{\bar{\mathcal{H}}'_i}, s_{\mathcal{H}(i,v_i)}, s_{\mathcal{H}'(i,v_i)}, s_{\bar{r}_i}, s_{\bar{r}'_i}, s_{r_{v_i}}, s_{r'_{v_i}}, s_{u_i}, s_{u'_i}\}_{i\in\gamma})$ where $\{s_{r_{i,1}} = r_{r_{i,1}} + c\cdot r_{i,1}, s_{r'_{i,1}} = r_{r'_{i,1}} + c\cdot r'_{i,1}, s_{r_{i,2}} = r_{r_{i,2}} + c\cdot r_{i,2}, s_{r'_{i,2}} = r_{r'_{i,2}} + c\cdot r'_{i,2}, s_{\bar{\mathcal{H}}_i} = r_{\bar{\mathcal{H}}_i} + c\cdot \bar{\mathcal{H}}_i, s_{\bar{\mathcal{H}}'_i} = r_{\bar{\mathcal{H}}'_i} + c\cdot \bar{\mathcal{H}}'_i, s_{\mathcal{H}(i,v_i)} = r_{\mathcal{H}(i,v_i)} + c\cdot \mathcal{H}(i,v_i), s_{\mathcal{H}'(i,v_i)} = r_{\mathcal{H}'(i,v_i)} + c\cdot \mathcal{H}'(i,v_i), s_{\bar{r}_i} = r_{\bar{r}_i} + c\cdot \bar{r}_i, s_{\bar{r}'_i} = r_{\bar{r}'_i} + c\cdot \bar{r}'_i, s_{r_{v_i}} = r_{r_{v_i}} + c\cdot r_{v_i}, s_{r'_{v_i}} = r_{r'_{v_i}} + c\cdot r'_{v_i}, s_{u_i} = r_{u_i} + c\cdot u_i, s_{u'_i} = r_{u'_i} + c\cdot u'_i\}_{i\in\gamma}$.

   $\mathcal{U}$ forwards $\{e_i, e'_i\}_{i\in\gamma}$, $\mathcal{T}_\gamma$, π^c, σ to $\mathcal{DO}$

3. $\mathcal{DO}$ verifies σ on $\mathcal{T}_\gamma$ using $pk_{\mathcal{CA}}$ and verifies π^c by

   (a) computing $\{\hat{e}_i = e_i^{-c} e_t^{s_{r_{i,1}}} \cdot \mathsf{h}^{s_{\bar{\mathcal{H}}_i}} (\mathsf{h}^q)^{s_{r_{i,2}}} \cdot \mathsf{g}^{s_{\bar{r}_i}}, \hat{e}'_i = e_i'^{-c} e_{t'_i}^{s_{r'_{i,1}}} \cdot \mathsf{h}^{s_{\bar{\mathcal{H}}'_i}} \cdot (\mathsf{h}^q)^{s_{r'_{i,2}}} \cdot \mathsf{g}^{s_{\bar{r}'_i}}, \hat{c}_{v_i} = c_{v_i}^{-c} \mathfrak{g}^{s_{\mathcal{H}(i,v_i)}} \mathfrak{h}^{s_{r_{v_i}}}$,
   $\hat{c}'_{v_i} = c'_{v_i}{}^{-c} \mathfrak{g}^{s_{\mathcal{H}'(i,v_i)}} \mathfrak{h}^{s_{r'_{v_i}}}, \hat{O}_{i,1} = O_{i,1}^{-c} c_{v_i}^{s_{r_{i,1}}} (1/\mathfrak{g})^{s_{\bar{\mathcal{H}}_i}} \mathfrak{h}^{s_{u_i}}, \hat{O}_{i,2} = O_{i,2}^{-c} c'_{v_i}{}^{s_{r'_{i,1}}} (1/\mathfrak{g})^{s_{\bar{\mathcal{H}}'_i}} \mathfrak{h}^{s_{u'_i}}\}_{i\in\gamma}$

   (b) checking if $c \stackrel{?}{=} \mathcal{H}(\mathsf{g}||\mathsf{h}||\mathfrak{g}||\mathfrak{h}||\mathcal{V}(e_i)||\mathcal{V}(e'_i)||\mathcal{V}(c_{v_i})||\mathcal{V}(c'_{v_i})||\mathcal{V}(\hat{e}_i)||\mathcal{V}(\hat{e}'_i)||\mathcal{V}(\hat{c}_{v_i})||\mathcal{V}(\hat{O}_{i,1}) ||\mathcal{V}(\hat{O}_{i,2}))$ where $\mathcal{V}(x_i) = x_{k_1}||\cdots||x_{k_j}||\cdots||x_{k_{|\gamma|}}$ in which $k_j \in \gamma$.

   (c) and checking if $\{s_{r_{i,1}}, s_{r'_{i,1}} \stackrel{?}{\in} \{0,1\}^{l_q+l_\phi+l_\mathcal{H}+1}\}_{i\in\gamma}$, $\{s_{r_{i,2}}, s_{r'_{i,2}} \stackrel{?}{\in} \{0,1\}^{l_q+2l_\phi+l_\mathcal{H}+1}\}_{i\in\gamma}$

   $\mathcal{DO}$ starts to define a polynomial $Q_x(\cdot)$ of degree $k_x - 1$ for each node x in $\mathcal{T}_\gamma$ in a top-down manner. For the root node r, it sets $Q_r(0) = y$ and $k_r - 1$ other points of Q_r randomly to fully define Q_r. For any other node x, set $Q_x(0) = Q_{parent(x)}(index(x))$ and choose $k_x - 1$ other points randomly to completely define Q_x. $\mathcal{DO}$ defines another polynomial $Q'_x(\cdot)$ in the same way as $Q_x(\cdot)$ except that $Q'_x(0) = y'$. For each $i \in \gamma$, $\mathcal{DO}$ decrypts $d_i = \mathsf{Dec}^{hom}_{sk_{\mathcal{DO}}}(e_i), d'_i = \mathsf{Dec}^{hom}_{sk_{\mathcal{DO}}}(e'_i)$ and sends $a_i = g^{Q_{node(\alpha_i)}(0)/d_i}$ and $a'_i = g^{Q'_{node(\alpha_i)}(0)/d_i}$ to $\mathcal{U}$.

4. $\mathcal{U}$ computes decryption key $sk_i = a_i{}^{r_{i,1}} = g^{Q_{node(\alpha_i)}(0)/(t+\mathcal{H}(i,v_i))}$ and search token $tk_i = a_i'^{r'_{i,1}} = g^{Q'_{node(\alpha_i)}(0)/(t+\mathcal{H}(i,v_i))}$ for $i \in \gamma$.

   $\mathcal{U}$ verifies sk_i-s as follows

   (a) compute $p_i = e(sk_i, T\cdot g^{\mathcal{H}(i,v_i)}) = e(g,g)^{Q_{node(\alpha_i)}(0)}$ for all $i \in \gamma$.

   (b) it starts to compute a value q_x for each node x in $\mathcal{T}_\gamma$ in a down-top manner starting from leaves. For each leaf node x in $\mathcal{T}_\gamma$, its q_x is set to $p_{attr(x)}$. For a non-leaf node x, q_x is dependent on k_x. If $k_x = 1$, user first verifies that each $q_{child_i(x)}$, for all $i \in S_x$, is the same. Then it sets $q_x = q_{child_i(x)}$, for arbitrary $i \in S_x$. If $k_x > 1$, it sets $q_x = \prod_{i\in S_x} (q_{child_i(x)})^{\Delta_{i,S_x}}$.

   (c) The procedure continues until it reaches the root node r. Finally, the user checks whether $q_r \stackrel{?}{=} Y$.

   $\mathcal{U}$ verifies tk_i as it does sk_i except that q_r should be equal to Y' this time.

Figure 3 cont.: The **AuthorizedBlindExtract** protocol

values $s, s' \in_R \mathbb{Z}_q$ and outputs the ciphertext as:

$$C = \left(E, E', \{E_i, E'_i\}_{i\in\Omega}\right).$$

where $E = \mathsf{Enc}^{sym}_{Y^s}(m)$, $E' = Y'^{s'}$, $E_i = g^{s\cdot(t+\mathcal{H}(i,v_i))}$ and $E'_i = g^{s'\cdot(t'+\mathcal{H}'(i,v_i))}$.

Extract($\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{DO}(params, msk_{\mathcal{DO}})$): This is an interactive protocol between $\mathcal{U}$ and $\mathcal{DO}$.

1. $\mathcal{U}$ chooses an attribute set γ and constructs $\mathcal{T}_\gamma$ and $\mathbf{v}_\gamma$ to fully define a conditional expression it

wants to query. Then it submits $\mathcal{T}_\gamma$ and $\mathbf{v}_\gamma$ to $\mathcal{DO}$.

2. $\mathcal{DO}$ defines a polynomial $Q_x(\cdot)$ of degree $k_x - 1$ for each node x in $\mathcal{T}_\gamma$ in a top-down manner. For the root node r, it sets $Q_r(0) = y$ and $k_r - 1$ other points of Q_r randomly to fully define $Q_r(\cdot)$. For any other node x, it sets $Q_x(0) = Q_{parent(x)}(index(x))$ and chooses $k_x - 1$ other points randomly to completely define $Q_x(\cdot)$. Then it outputs decryption key $sk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)} = \{\{sk_i\}_{i\in\gamma}, \mathcal{T}_\gamma, \mathbf{v}_\gamma\}$ where $sk_i = g^{Q_{node(\alpha_i)}(0)/(t+\mathcal{H}(i,v_i))}$. $\mathcal{DO}$ defines $Q'_x(\cdot)$ in the same way as $Q_x(\cdot)$ except that $Q'_r(0) = y'$. And it outputs search token $tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)} = \{\{tk_i\}_{i\in\gamma}, \mathcal{T}_\gamma\}$ where $tk_i = g^{Q'_{node(\alpha_i)}(0)/(t'+\mathcal{H}'(i,v_i))}$. Last, $\mathcal{DO}$ sends $tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}$ and $sk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}$ to $\mathcal{U}$.

Test($\mathcal{S}(params, tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}, C)$**)**: To test whether an encrypted record $C = \mathsf{Encrypt}(msk_{\mathcal{DO}}, \mathbf{v}'_\Omega)$ matches a search token $tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)} = \{\{tk_i = g^{Q'_{node(\alpha_i)}(0)/(t'+\mathcal{H}'(i,v_i))}\}_{i\in\gamma}, \mathcal{T}_\gamma\}$, it first calculates $\Gamma_{\mathcal{T}_\gamma}$ from $\mathcal{T}_\gamma$. The search operation starts from the first $\gamma' \in \Gamma_{\mathcal{T}_\gamma}$. Let $i = attr(x)$. For each node x in $\mathcal{T}_{\gamma'}$, it computes a value z_x in a down-top manner. For each leaf node x in $\mathcal{T}_{\gamma'}$, $\mathcal{S}$ computes $z_x = \hat{e}(tk_i, E'_i)$. We use v'_i to denote the value embedded in E'_i. Note if $v_i = v'_i$, $z_x = \hat{e}(g^{Q'_x(0)/(t'+\mathcal{H}'(i,v_i))}, g^{s'\cdot(t'+\mathcal{H}'(i,v'_i))}) = \hat{e}(g,g)^{s'\cdot Q'_x(0)}$. For each non-leaf node x, it sets $z_x = \prod_{i\in S_x}(z_{child_i(x)})^{\Delta_{i,S_x}}$. Note if $\{v_i = v'_i\}_{i\in\gamma'}$, $z_x = \prod_{i\in S_x}(\hat{e}(g,g))^{s'\cdot Q'_{child_i(x)}(0)\cdot\Delta_{i,S_x}}$ $= \prod_{i\in S_x}(\hat{e}(g,g))^{s'\cdot Q'_x(i)\cdot\Delta_{i,S_x}} = \hat{e}(g,g)^{s'\cdot Q'_x(0)}$. The procedure continues until it reaches the root node r. If $z_r = E'$, $\mathcal{S}$ outputs 'yes'. Otherwise, it continues to test the next γ'. If all γ's do not meet the criteria, it outputs 'no'.

Decrypt($\mathcal{U}(params, tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}, sk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}, C)$**)**: The decryption algorithm first identifies γ' satisfying $tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)}$ as **Test** algorithm does. Note this step can be omitted if γ' is provided as input after it is identified by **Test**. Then it follows a down-top manner in $\mathcal{T}_{\gamma'}$. Let $i = attr(x)$. Then for each leaf node $x \in \mathcal{T}_{\gamma'}$, it computes $z_x = \hat{e}(sk_i, E_i)$. Note since v_i equals to v'_i, $z_x = \hat{e}(g^{Q_x(0)/(t_i+t\cdot v_i)}, g^{s(t_i+t\cdot v'_i)}) = \hat{e}(g,g)^{s\cdot Q_x(0)}$. For non-leaf node $x \in \mathcal{T}_{\gamma'}$, it computes $z_x = \prod_{i\in S_x}(z_{child_i(x)})^{\Delta_{i,S_x}} = \prod_{i\in S_x}(\hat{e}(g,g))^{s\cdot Q_{child_i(x)}(0)\cdot\Delta_{i,S_x}} = \prod_{i\in S_x}(\hat{e}(g,g))^{s\cdot Q_x(i)\cdot\Delta_{i,S_x}} = \hat{e}(g,g)^{s\cdot Q_x(0)}$. The procedure continues until it reaches root r and $z_r = \hat{e}(g,g)^{s\cdot Q_r(0)} = \hat{e}(g,g)^{s\cdot y} = Y^s$ is computed. Then user recovers $m = \mathsf{Dec}^{sym}_{\mathcal{H}(Y^s)}(E)$.

BlindExtract($\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{DO}(params, msk_{\mathcal{DO}})$**)**:

1. $\mathcal{U}$ first verifies π^s. If π^s passes verification, then the user chooses $r_{i,1}, r'_{i,1} \in_R \mathbb{Z}_q$ and $r_{i,2}, r'_{i,2} \in_R [0,\ldots,2^k q]$ and computes

$$e_i = ((e_t \oplus \mathsf{Enc}^{hom}_{pk_s}(\mathcal{H}(i,v_i))) \otimes r_{i,1}) \oplus \mathsf{Enc}^{hom}_{pk_s}(r_{i,2}\cdot q), \forall i \in \gamma$$

$$e'_i = ((e_{t'} \oplus \mathsf{Enc}^{hom}_{pk_s}(\mathcal{H}'(i,v_i))) \otimes r'_{i,1}) \oplus \mathsf{Enc}^{hom}_{pk_s}(r'_{i,2}\cdot q), \forall i \in \gamma$$

It also computes a zero-knowledge proof π^c proving e_i, e'_i are well formed and $r_{i,1}, r_{i,2}, r'_{i,1}, r'_{i,2}$ are in appropriate interval. Then it sends $\{e_i, e'_i\}_{i\in\gamma}$, $\mathcal{T}_\gamma$, π^c to $\mathcal{DO}$.

2. $\mathcal{DO}$ verifies π^c to make sure $e_i, e'_i, r_{i,1}, r'_{i,1}, r_{i,2}, r'_{i,2}$ are correctly embedded. Then $\mathcal{DO}$ starts to define a polynomial $Q_x(\cdot)$ of degree $k_x - 1$ for each node x in $\mathcal{T}_\gamma$ in a top-down manner. For the root node r, it sets $Q_r(0) = y$ and $k_r - 1$ other points of Q_r randomly to fully define Q_r. For any other node x, set $Q_x(0) = Q_{parent(x)}(index(x))$ and choose $k_x - 1$ other points randomly to completely define Q_x. $\mathcal{DO}$ defines another polynomial $Q'_x(\cdot)$ in the same way as $Q_x(\cdot)$ except that $Q'_x(0) = y'$. Next, for each $i \in \gamma$, $\mathcal{DO}$ decrypts $d_i = \mathsf{Dec}^{hom}_{sk_{\mathcal{DO}}}(e_i), d'_i = \mathsf{Dec}^{hom}_{sk_{\mathcal{DO}}}(e'_i)$ and sends $a_i = g^{Q_{node(\alpha_i)}(0)/d_i}$ and $a'_i = g^{Q'_{node(\alpha_i)}(0)/d'_i}$ to $\mathcal{U}$.

3. $\mathcal{U}$ computes $sk_i = a_i^{r_{i,1}} = g^{Q_{node(\alpha_i)}(0)/(t+\mathcal{H}(i,v_i))}$ and $tk_i = a_i'^{r'_{i,1}} = g^{Q'_{node(\alpha_i)}(0)/(t'+\mathcal{H}'(i,v_i))}$ for $i \in \gamma$. Then $\mathcal{U}$ checks the validity of sk_is. To do that, it computes $p_i = e(sk_i, T \cdot g^{\mathcal{H}(i,v_i)}) = e(g,g)^{Q_{node(\alpha_i)}(0)}$ for all $i \in \gamma$. After that, it starts to compute a value q_x for each node x in $\mathcal{T}_\gamma$ in a down-top manner starting from leaves. For each leaf node x in $\mathcal{T}_\gamma$, its q_x is set to $p_{attr(x)}$. For a non-leaf node x, q_x is dependent on k_x. If $k_x = 1$, user first verifies that each $q_{child_i(x)}$, for all $i \in S_x$, is the same. Then it sets $q_x = q_{child_i(x)}$, for arbitrary $i \in S_x$. If $k_x > 1$, it sets $q_x = \prod_{i \in S_x}(q_{child_i(x)})^{\Delta_{i,S_x}}$. The procedure continues until it reaches the root node r. Finally, the user checks whether $q_r \overset{?}{=} Y$. If any above verification fails, $\mathcal{U}$ quits. $\mathcal{U}$ checks tk_i in the same way as it does sk_i except that q_r should be equal to Y' this time. $\mathcal{U}$ outputs decryption key $sk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)} = \{\{sk_i\}_{i\in\gamma}, \mathcal{T}_\gamma, \mathbf{v}_\gamma\}$ and search token $tk_{(\mathcal{T}_\gamma,\mathbf{v}_\gamma)} = \{\{tk_i\}_{i\in\gamma}, \mathcal{T}_\gamma\}$.

Authorize($\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma), \mathcal{CA}(params, sk_{\mathcal{CA}})$): $\mathcal{U}$ submits $\mathcal{T}_\gamma, \mathbf{v}_\gamma$ to $\mathcal{CA}$. $\mathcal{CA}$ verifies that $\mathcal{U}$ has the right to search for the conditional expression defined by $(\mathcal{T}_\gamma, \mathbf{v}_\gamma)$. If it approves user request, then $\mathcal{CA}$, on $\mathcal{U}$'s behalf, makes pedersen commitments c_{v_i}, c'_{v_i} on each $v_i \in \mathbf{v}_\gamma$, i.e. $c_{v_i} = \mathfrak{g}^{\mathcal{H}(i,v_i)}\mathfrak{h}^{r_{v_i}}$ and $c'_{v_i} = \mathfrak{g}^{\mathcal{H}'(i,v_i)}\mathfrak{h}^{r'_{v_i}}$. Next, $\mathcal{CA}$ maps $\mathcal{T}_\gamma$ to a Merkle hash tree. Specifically, it computes a hash value for each node x in $\mathcal{T}_\gamma$. For each leaf node x, its hash value is $h_x = \mathcal{H}(k_x)$. For non-leaf node, its hash value is defined as the hash of concatenations of its k_x and its children's hash values, i.e. $h_x = \mathcal{H}(k_x || h_{child_1(x)} || \cdots || h_{child_{num_x}(x)})$. Let h_r denote the hash value for the root node r. CA issues a signature σ on h_r and $\{c_{v_i}, c'_{v_i}\}_{i\in\gamma}$, i.e. $\sigma = \mathsf{Sign}_{sk_{\mathcal{CA}}}(h_r, \{c_{v_i}, c'_{v_i}\}_{i\in\gamma})$, and sends $\{\{r_{v_i}, c_{v_i}, r'_{v_i}, c'_{v_i}\}_{i\in\gamma}, \sigma\}$ back to $\mathcal{U}$.

AuthorizedBlindExtract($\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma, \psi, open, \sigma), \mathcal{DO}(params, msk_{\mathcal{DO}})$): This protocol is detailed in Fig. 3. Here $\psi = \{c_{v_i}, c'_{v_i}\}_{i\in\gamma}$ and $open = \{r_{v_i}, r'_{v_i}\}_{i\in\gamma}$. The protocol basically follows the **BlindExtract** protocol except that $\mathcal{U}$ needs to prove statements about commitments using zero-knowledge proof.

6 Performance Analysis

Before presenting performance analysis, we point out two possible improvements to the scheme. First, in **Test** algorithm, if the identified matching set γ' is sent to $\mathcal{U}$, then **Decrypt** algorithm does not need search token to seek γ' again. Second, as pointed out in [20], instead of exponentiating at each level during the computation of z_x in **Decrypt**, for each leaf node in γ', we can keep track of which Lagrange coefficient is multiplied with each other. Using this, we can compute the final exponent f_x for each leaf node $x \in \mathcal{T}_{\gamma'}$ by doing multiplication in $\mathbb{Z}_q$. Now z_r is simply $\prod_{i\in\gamma'} \hat{e}(sk_i, E_i)^{f_{node(\alpha_i)}}$. The same optimization applies to **Test** algorithm.

We now consider the efficiency of the scheme. The **Encrypt** algorithm takes $2n$ group exponentiations in $\mathbb{G}_1$. The **Extract** algorithm takes $2\cdot|\gamma|$ group exponentiations in $\mathbb{G}_1$. In **BlindExtract** algorithm, $\mathcal{DO}$ spends $20\cdot|\gamma|$ group exponentiations in $\mathbb{G}_1$. $\mathcal{U}$ spends $28\cdot|\gamma|$ group exponentiations in $\mathbb{G}_1$ plus some verification time dependent on access tree. The **Test** algorithm's performance depends on the access tree $\mathcal{T}_\gamma$. In conjunction-only case, it involves 1 test of $|\gamma|$ pairing and $|\gamma|$ exponentiation in $\mathbb{G}_2$. In disjunction-only case, it involves $|\gamma|$ tests of 1 pairing operation. Compared to $|\gamma|$ pairing overhead in [10,22,32], our scheme has similar overhead while supporting more flexible queries. The optimized **Decrypt** algorithm takes $|\gamma'|$ pairing and $|\gamma'|$ group exponentiations in $\mathbb{G}_2$.

7 Performance Evaluation

We implemented the proposed scheme in C++ using PBC (ver. 0.57) [24] and OpenSSL (ver. 1.0.0) [1] library. This section discusses the performance of each function in our scheme. All benchmarks were

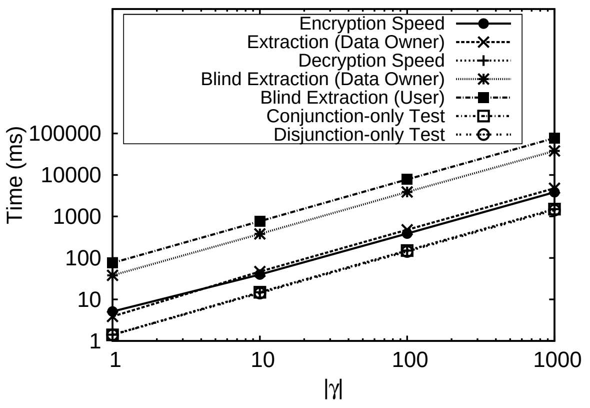


Figure 4: Performance of **Encryp**, **Extract**, **Decrypt** vs. number of attributes.

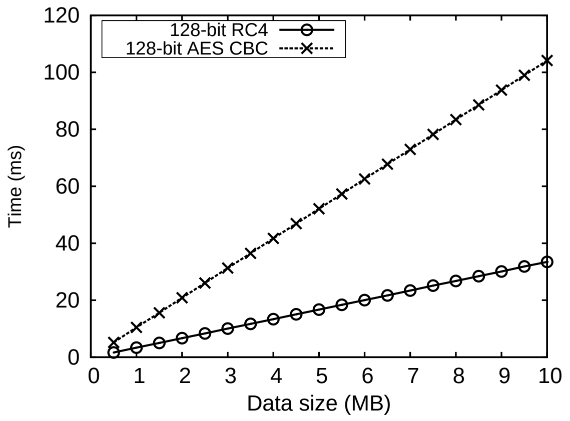


Figure 5: Symmetric encryption overhead.

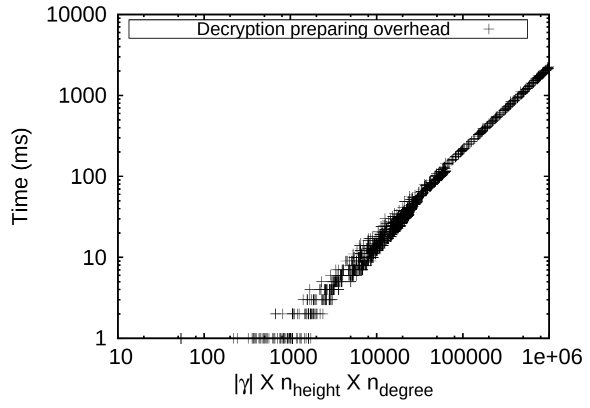


Figure 6: Decryption preparing time.

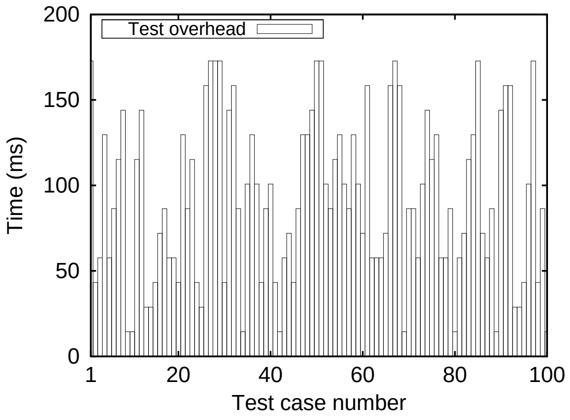


Figure 7: Performance of **Test** when $|\gamma| = 10$.

performed on a Ubuntu 9.10 desktop platform with Intel Core i7-920 (2.66GHz and 8MB cache) and 6GB RAM.

Since performance of each function only depends on the access tree, we do not consider the performance impact of the contents associated with leaf nodes. We use a random access tree (in all tests) that is generated as follows. First we fix the number of leaves, n_{leaves}. Then a random tree height n_{height} between 1 and 5 is chosen. The node degree is computed as $n_{degree} = \lceil n_{leaves}^{1/n_{leaves}} \rceil$. After $n_{leaves}, n_{height}, n_{degree}$ is determined, the random tree is constructed in a down-top manner. At depth l, one parent node is constructed for every n_{degree} nodes at depth $l+1$. If less than n_{degree} nodes are left at depth $l+1$, one parent node is constructed for these remaining nodes. The procedure continues until only one parent (root) can be constructed. For simplicity, we assume the total number of attributes $w = |\gamma| = n_{leaves}$.

First we test the speed of **Encrypt**. Fig. 4 (Encryption Speed line) shows the overhead to compute $Y^s, E', \{E_i, E_i'\}_{i \in \Omega}$ versus the number of attributes $|\gamma|$. As we can see, its overhead increases linearly with $|\gamma|$. Fig. 5 shows the performance of symmetric encryption, which is needed to compute $E = \mathsf{Enc}^{sym}_{\mathcal{H}(Y^s)}(m)$.

Extract and **BlindExtract** performance is also shown in Fig. 4. In this test, the threshold gates in the access tree are chosen randomly. The overhead of **Extract** (Extraction (Data Owner) line) is solely at $\mathcal{DO}$ side and it increases linearly with $|\gamma|$. The overhead of **BlindExtract** is at both $\mathcal{U}$ side and $\mathcal{DO}$

side. The overhead at $\mathcal{DO}$ side (Blind Extraction (Data Owner) line) is almost nine times that of normal extraction. The overhead at $\mathcal{U}$ side (Blind Extraction (User) line) doubles that at $\mathcal{DO}$ side.

To test **Decrypt**, we assume $\gamma' = \gamma$, i.e., all attributes should be involved in the decryption. Since all threshold gates in $\mathcal{T}_{\gamma'}$ should be conjunctive gates, we make them conjunctive in the random access tree $\mathcal{T}_\gamma$ as well. Fig. 4 (Decryption Speed line) shows the speed to recover Y^s. We find that decryption overhead increases linearly with $|\gamma|$ and it is even cheaper than extraction. The reason is because pairing operation and exponentiation in $\mathbb{G}_2$ is faster than exponentiation in $\mathbb{G}_1$[1]. Fig. 6 shows the speed of computing f_x for all leaf node x, which is necessary for the optimization of decryption. Its speed is almost linear with the product of $|\gamma|$, tree height and tree degree. Note this part of operation can be conducted offline and only needs to be computed once for one type of access tree. The performance of $\mathsf{Dec}^{sym}_{\mathcal{H}(Y^s)}(E)$ is same as $\mathsf{Enc}^{sym}_{\mathcal{H}(Y^s)}(m)$ as shown in Fig. 5.

As to **Test** performance, it highly depends on the access tree. During the following test, the performance is recorded in the worst case, i.e. all possible subtrees $\mathcal{T}_{\gamma'}$ of $\mathcal{T}_\gamma$ are tried. Fig. 4 shows the conjunction-only **Test** and disjunction-only **Test** performance. As we can see, they all increase linearly with $|\gamma|$. The reason why they are almost the same is because conjunction-only **Test** has 1 test involving $|\gamma|$ pairing and $|\gamma|$ exponentiation in $\mathbb{G}_2$ while disjunction-only **Test** has $|\gamma|$ tests involving 1 pairing. To further test **Test** operation, we use random access tree. We restrict $|\gamma|$ to be 10, which is usually enough for normal query, and set each threshold gate in the tree randomly. Fig. 7 shows the results of 100 test cases. As we can see the maximum **Test** time is 170ms and the average **Test** time is 85ms. In cloud computing scenario, multiple **Test** operations can run simultaneously and therefore spending average 85ms on each record is acceptable.

8 Limitation

The proposed scheme has some limitation and it should be considered in future work. First, it only supports equality testing. Practical privacy-preserving comparison is not available yet. Second, it only hides concrete value in the conditional expression and the structure $\mathcal{T}_\gamma$ is revealed to the adversary. Third, join operations between two tables are not supported. Fourth, if the set of possible attribute values in γ is small, the adversary can always try to encrypt something under all possible values and run **Test** over the encryptions to see if there is a match. This would reveal v_γ within $tk_{(\mathcal{T}_\gamma, \mathbf{v}_\gamma)}$. However, the complexity of such brute force attacks against this intrinsic weakness of public key-based searchable encryption, grows exponentially with $|\gamma|$. Fifth, $\mathcal{DO}$ is required to be online to help $\mathcal{U}$ extract search tokens and decryption keys. However, we expect that this functionality can be finished by some secure hardware that can be safely installed at $\mathcal{U}$ side without compromising $msk_{\mathcal{DO}}$.

9 Conclusion

This paper provides an overview of privacy challenges facing cloud storage and develops a novel encryption scheme for coping with these challenges. The scheme hides the plaintext of database and user's query content from the cloud server. It allows data owner to do content-level fine-grained access control by issuing users appropriate search tokens and decryption keys. The scheme also supports blind retrieval of search tokens and decryption keys in the sense that neither data owner nor cloud server learns the query content. The additional feature of user input authorization by CA can also be supported. Our evaluation shows that the proposed scheme's performance falls within the acceptable range.

[1]In our benchmark of Type A pairing family in [24], one exponentiation in $\mathbb{G}_1$ takes 1.9 ms, one exponentiation in $\mathbb{G}_2$ takes 0.18 ms while one group pairing takes 1.4 ms.

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A Security Proof

The following three lemmas establish the security of our scheme.

Lemma 1. *The scheme* Π *is Selective-Set Secure (Def. 1) under the BDH assumption.*

Proof. We prove our scheme is Selective-Set Secure through a series of game reductions. Suppose $(E, E', \{E_i, E'_i\}_{i\in\Omega})$ is an encryption of message m. We use $Game_0$ to denote the original scheme. In $Game_1$, we replace $t + \mathcal{H}(i, v_i)$ with a random number. In $Game_2$, we replace E and each E_i with independent random numbers from $Game_1$. In $Game_3$, we replace E' and each E'_i with independent random numbers from $Game_2$. We use symbol $Game_i \approx Game_{i+1}$ to denote that the view to $\mathcal{A}$ in $Game_i$ and $Game_{i+1}$ are indistinguishable. Our goal is to show that $Game_0 \approx Game_1 \approx Game_2 \approx Game_3$ if $\mathcal{A}$ never queries $\mathcal{O}_{\mathsf{Extract}}(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ such that $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m) = 1$. The following argument establishes the indistinguishability between each two immediate games.

$Game_0 \approx Game_1$: It is obvious that $Game_0$ and $Game_1$ are indistinguishable because $\mathcal{H}(i, v_i)$ can always return random values for each random oracle query.

$Game_1 \approx Game_2$: In order to show that $Game_1$ and $Game_2$ are indistinguishable, we need to rely on BDH assumption. Given a BDH challenge (A, B, C, Z), the reducation algorithm sets $Y = \hat{e}(A, B) = \hat{e}(g, g)^{ab}$ and $E = \mathsf{Enc}_Z^{sym}(m)$. For each $i \in \Omega$, it picks r_i and sets $E_i = C^{r_i}$. Let $s = c$ so we have $Y^s = (\hat{e}(g, g)^{ab})^c = \hat{e}(g, g)^{abc}$. When $(A, B, C, Z) = (g^a, g^b, g^c, \hat{e}(g, g)^{abc})$, we have $Y^s = Z$ and $E_i = (g^{r_i})^s$. Therefore the view to $\mathcal{A}$ in this case is equivalent to $Game_1$. When $(A, B, C, Z) = (g^a, g^b, g^c, \hat{e}(g, g)^z)$, E and E_i become independent random values. Therefore the view to $\mathcal{A}$ in this case is equivalent to $Game_2$.

One remaining part for the above argument is to show that the reducation algorithm is able to answer $\mathcal{O}_{\mathsf{Extract}}(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ when $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m) \neq 1$. Since $E', \{E'_i\}_{i\in\Omega}$ are not replaced in $Game_1$, $tk_{\mathcal{T}_\gamma,\mathbf{v}_\gamma}$ can still be generated following normal operations in Extract($\mathcal{U}(params, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$, $\mathcal{DO}(params, msk_{\mathcal{DO}})$). To generate $sk_{\mathcal{T}_\gamma,\mathbf{v}_\gamma}$, reduction needs to assign a polynomial Q_x of degree $d_x = k_x - 1$ for every node in $\mathcal{T}_\gamma$ such that $Q_r(0) = y = ab$. We make $Q_x(\cdot) = b \cdot q_x(\cdot)$ and start to define $q_x(\cdot)$ instead. Note $q_r(0) = a$. We first define the following two procedures: *CoKnown* and *CoUnknown*.

CoKnown(x, λ_x) This procedure sets up the polynomial for the nodes of an access subtree with root node x and $q_x(0) = \lambda_x$. Note, in this procedure, λ_x is known.

It first sets up a polynomial q_x of degree d_x for the root node x. It sets $q_x(0) = \lambda_x$ and then sets the rest of the points randomly to completely fix q_x. Now it sets polynomials for each child node x' of x by calling the procedure *CoKnown*$(x', q_x(index(x')))$. Notice that in this way, $q_{x'}(0) = q_x(index(x'))$ for each child node x' of x.

CoUnknown(x, λ_x) This procedure sets up the polynomials for the nodes of an access subtree with root node x. Note, in this procedure, λ_x is unknown but g^{λ_x} is known. Therefore this procedure makes $g^{q_x(0)} = g^{\lambda_x}$.

It first defines a polynomial q_x of degree d_x for the root node x such that $g^{q_x(0)} = g^{\lambda_x}$. Because the access subtree is unsatisfied, no more than d_x children of x are satisfied. Let $h_x \leq d_x$ be the number of satisfied children of x. For each satisfied child x' of x, the procedure chooses a random point $\lambda_{x'} \in \mathbb{Z}_p$ and sets $q_x(index(x')) = \lambda_{x'}$. It then fixes the remaining $d_x - h_x$ points of q_x randomly to completely define q_x. Now the algorithm recursively defines polynomials for the rest of the nodes in the tree as follows. For each child node x' of x, the algorithm calls:

CoKnown$(x', q_x(index(x')))$ if x' is a satisfied node. Notice that $q_x(index(x'))$ is known in this case.

CoUnknown$(x', q_x(index(x'))$ if x' is not a satisfied node. Notice that only $g^{q_x(index(x'))}$ can be obtained by interpolation as only $g^{q_x(0)}$ is known in this case.

To define $q_x(\cdot)$ for each node x of $\mathcal{T}_\gamma$, reduction runs *CoUnknown*(r, a) where r is the root node of $\mathcal{T}_\gamma$ and $g^a = A$. Note, for each leaf node x, $q_x(0)$ is known if x is satisfied and $g^{q_x(0)}$ is known if x is unsatisfied. Now we define $Q_x(\cdot) = b \cdot q_x(\cdot)$. Therefore, The key corresponding to a satisfied node is $D_x = g^{\frac{Q_x(0)}{r_i}} = g^{\frac{bq_x(0)}{r_i}} = B^{\frac{q_x(0)}{r_i}}$. For unsatisfied node, we choose a random number $\beta_i \in \mathbb{Z}_p$ and make $r_i = b\beta_i$. Then the key corresponding to an unsatisfied node is $D_x = g^{\frac{Q_x(0)}{r_i}} = g^{\frac{bq_x(0)}{b\beta_i}} = g^{\frac{q_x(0)}{\beta_i}}$. This finishes the procedure for answering $\mathcal{O}_{\mathsf{Extract}}(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$.

In sum, differentiating $Game_1$ from $Game_2$ is equivalent to breaking BDH assumption and thus we have $Game_1$ and $Game_2$ are indistinguishable.

$Game_2 \approx Game_3$: This reduction is similar to the reduction from $Game_1$ to $Game_2$ except that $E', \{E'_i\}$ are replaced this time.

Going back to the Def. 1, since c^* can be replaced with random values without presenting distinguishable views to $\mathcal{A}$ as long as $\mathcal{A}$ not querying $\mathcal{O}_{\mathsf{Extract}}(params, msk_{\mathcal{DO}}, \mathcal{T}_\gamma, \mathbf{v}_\gamma)$ with $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m_1) = 1$ or $\mathcal{T}_\gamma(\mathbf{v}_\gamma, m_2) = 1$. Therefore the advantage of $\mathcal{A}$ in winning the Selective-Set Secure game is negligible.

□

Lemma 2. BlindExtract *is a* leak-free *(Def. 2) protocol.*

Proof. To show that **BlindExtract** is leak-free. We should construct a simulator that talks to a trusted party to obtain $\{sk_i, tk_i\}_{i\in\gamma}$ and simulates to the malicious user. The simulator is constructed as follows:

1. Simulator runs **Setup**(1^k) and publishes e_t as $\mathsf{Enc}^{hom}_{pk_{\mathcal{DO}}}(0)$, $e_{t'}$ as $\mathsf{Enc}^{hom}_{pk_{\mathcal{DO}}}(0)$.
2. Simulator receives $\{e_i, e'_i\}_{i\in\gamma}, \mathcal{T}_\gamma, \pi^c$ from $\mathcal{A}$ and verifies π^c to ensure $\{e_i, e'_i\}_{i\in\gamma}$ are computed correctly.
3. Simulator runs the zero-knowledge proof extraction algorithm and extracts $\{\mathcal{H}(i, v_i), \mathcal{H}'(i, v_i), r_{i,1}, r'_{i,1}\}_{i\in\gamma}$ from π^c.
4. Simulator submits $\mathcal{T}_\gamma, \{\mathcal{H}(i, v_i), \mathcal{H}'(i, v_i)\}_{i\in\gamma}$ to a trusted party which honestly computes and returns $\{sk_i, tk_i\}_{i\in\gamma}$ to the simulator.
5. Simulator returns $\{sk_i^{1/r_{i,1}}, tk_i^{1/r'_{i,1}}\}_{i\in\gamma}$ to $\mathcal{A}$.

Now we need to show that the view to $\mathcal{A}$ when talking to this simulator is indistinguishable from the view in the real game. We prove this through a series of games.

Game$_0$: This is essentially the real game.

Game$_1$: In this game, $\mathcal{A}$ interacts with a simulator S' that behaves the same as the final simulator from step 3 and behaves as the real protocol in step 1 and 2.

Game$_2$: This is essentially the final simulator. It only differs from the *Game*$_1$ in that $e_t, e_{t'}$ are generated by encrypting 0.

Game$_0$ is indistinguishable from *Game*$_1$ due to the soundness and extraction property of the zero-knowledge proof system. *Game*$_1$ is indistinguishable from *Game*$_2$ due to the semantic security of the encryption scheme. Therefore the probability that $\mathcal{A}$ can differentiate *Game*$_0$ from *Game*$_2$ is negligible. □

Lemma 3. **BlindExtract** *is a* selective-failure *(Def. 3) blind protocol.*

Proof. To show that **BlindExtract** is *selective-failure* blind, we first show that malicious server cannot learn useful information during the interaction with the user. We construct a simulator that simulates to the malicious server. The simulator is constructed as follows:

1. Simulator receives e_t and $e_{t'}$ from $\mathcal{A}$ and extracts t, t' from π^s.
2. Simulator chooses a random value $\{r_i, r'_i \in_R [0, 2^k q^2]\}_{i\in\gamma}$. It computes $\{e_i = \mathsf{Enc}^{hom}_{pk_{\mathcal{S}}}(r_i), e'_i = \mathsf{Enc}^{hom}_{pk_{\mathcal{S}}}(r'_i)\}_{i\in\gamma}$ and sends $\{e_i, e'_i\}_{i\in\gamma}$ to $\mathcal{A}$.
3. Simulator simulates the zero knowledge proof to $\mathcal{A}$.

Now we need to show that the view to $\mathcal{A}$ when talking to this simulator is indistinguishable from the view in the real game. We prove this through a series of games.

Game$_0$: This is essentially the real game.

Game$_1$: It is the same as *Game*$_0$ except that we use zero-knowledge simulation to do the proof in step 3.

Game$_2$: This is essentially the final simulator. It only differs from the *Game*$_1$ in that e_i, e'_i are generated by computing $\{e_i = \mathsf{Enc}^{hom}_{pk_{\mathcal{S}}}(r_i), e'_i = \mathsf{Enc}^{hom}_{pk_{\mathcal{S}}}(r'_i)\}_{i \in \gamma}$ instead of $e_i = ((e_t \oplus \mathsf{Enc}^{hom}_{pk_s}(\mathcal{H}(i, v_i))) \otimes r_{i,1}) \oplus \mathsf{Enc}^{hom}_{pk_s}(r_{i,2} \cdot q)$, $e'_i = ((e_{t'} \oplus \mathsf{Enc}^{hom}_{pk_s}(\mathcal{H}'(i, v_i))) \otimes r'_{i,1}) \oplus \mathsf{Enc}^{hom}_{pk_s}(r'_{i,2} \cdot q)$.

Game$_1$ is indistinguishable from *Game*$_0$ due to the property of zero-knowledge proof system. *Game*$_2$ is indistinguishable from *Game*$_1$ because $(t + \mathcal{H}(i, v_i)) \cdot r_{i,1} + r_{i,2} \cdot q$ is uniformly distributed over $[0, 2^k q^2]$, therefore indistinguishable from r_i. Similarly, $(t' + \mathcal{H}'(i, v_i)) \cdot r'_{i,1} + r'_{i,2} \cdot q$ is indistinguishable from r'_i. Therefore the probability that $\mathcal{A}$ can differentiate *Game*$_0$ from *Game*$_2$ is negligible.

Going back to the Def. 3, $\mathcal{A}$ can run one or both of the oracles up to the point $\{e_i, e'_i\}_{i \in \gamma}$ are received. Based on above argument, e_i, e'_i is indistinguishable from an encryption of random value distributed over $[0, 2^k q^2]$, and therefore the distribution of the two oracles are computationally indistinguishable. Now we show that $\mathcal{A}$ can predict the output s_0, s_1 without further interaction with the oracles:

1. $\mathcal{A}$ does the verification of step 3 in **BlindExtract**. If the verification fails, it records $s_0 = \perp$. Otherwise, $\mathcal{A}$ records $s_0 = \textbf{Extract}(\mathcal{U}(params, \mathcal{T}, \mathbf{v}_0), \mathcal{S}(params, msk_{\mathcal{S}}))$.

2. In turn, $\mathcal{A}$ records s_1 as it does for s_0.

3. Finally $\mathcal{A}$ predicts (s_0, s_1), if both $s_0 \neq \perp$ and $s_1 \neq \perp$; $\mathcal{A}$ predicts $(\varepsilon, \perp)$ if only $s_1 = \perp$; $\mathcal{A}$ predicts $(\perp, \varepsilon)$ if only $s_0 = \perp$; and $\mathcal{A}$ predicts $(\perp, \perp)$ if $s_0 = s_1 = \perp$.

These predictions result in the same distributions as those returned by the oracle, as the same checks are preformed. □